FaSMEd Case Study – DUE: „Can I sketch a graph based on a given situation?"

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This case study focuses on the development and implementation of a digital tool for formative self-assessment in mathematics. The aim is the design of a tool that allows students to become assessors themselves rather than having the technology evaluate their answers. Therefore, students are provided with a check-list that implies information on typical misconceptions in the field of functions to examine their own solution to an open assessment task. Based on this self-assessment, learners can choose to view more information on their previous mistakes and work on specific practice tasks. Chosen student cases show prospects and challenges of this approach.

I PORTFOLIO OF EVIDENCE

1- Tasks and resources used

The digital self-assessment tool "Can I sketch a graph based on a given situation?" was used in this case study. The tool is implemented in the software TI-Nspire Navigator and was used on iPads in the classroom.

Design and testing of the first version

The design of the tool is reclined to self-assessment material from the project KOSIMA (German acronym for: contexts for meaningful mathematics lessons, Barzel et al. 2011). The aim is to create a tool that allows students to become their own assessors rather than having the technology assess by counting the number of correct answers to a series of questions. This is why the tool centres around a self-check that helps students assess their own solution to an open assessment task. The mathematical content of the tool lies in the field of functions. It requires the learner to transfer a situational description of a functional relation into a graphical representation.

The first version of the tool was developed in a pen-and-paper version from May till November 2014 at the University of Duisburg-Essen. It consists of four different types of cards:
Test and *Check* (labelled with a magnifying glass and the numbers 1.1-1.2),
Good-to-know (labelled with a light bulb and the numbers 2.1-2.5),
Practice (labelled with a notebook and the numbers 3.1-3.8),
Expand (labelled with gearwheels and the number 4).

The hyperlink structure of these cards is shown in figure 1:
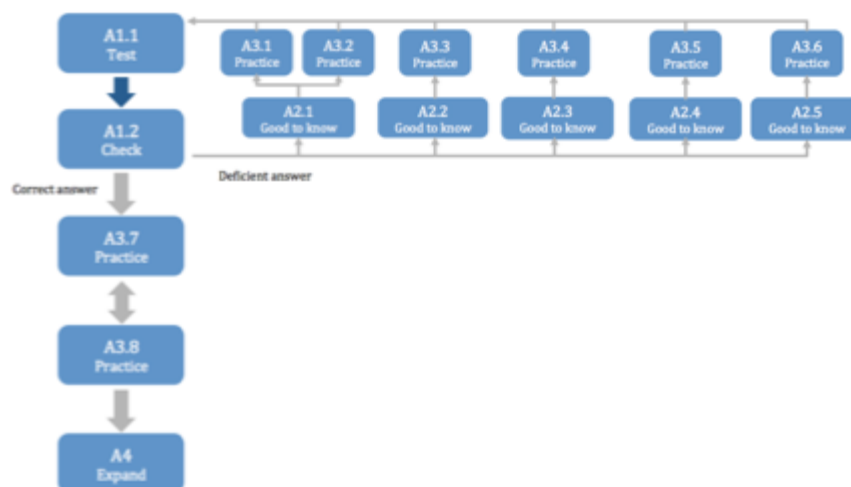

Figure 1: Structure of the first version of the self-assessment tool

A student begins by working on the open assessment task on the *Test*-card. The learner is given a story about a bike ride without specific values for the speed and is asked to draw a graph that shows how the speed changes as a function of the time. Then the student looks at two sample solutions and a list of criteria for successfully solving this task on the back of the card, before moving on to the *Check*-card. The self-check consists of ten statements that are formulated in a negative way, e.g. "My graph is not increasing at the beginning." This way, a student has to check off a statement to identify a mistake in his/her graph. If the task is solved correctly, the learner is referred to two more *Practice*-cards. Afterwards the learner is asked to continue with the *Expand*-card, that requires an extended amount of mathematical competencies to solve due to a more challenging context. If the student determines a mistake on the *Check*-card, he/she is referred to a *Good-to-know*-card and a *Practice*-card. It is the student's choice to get more information on his/her possible misconception in form of a *Good-to-know*-card that refers to the example of the *Test*-task and/or solve a task based on this particular issue. Afterwards, the open assessment task on the *Test*-card is repeated or revised and the student assesses his/her answer again.

(See the first pen-and-paper version of the tool in the appendix of this document.)

This first version of the tool was tested and reviewed from December 2014 until April 2015. There were three single student interviews in December 2014 in the Clara-Schumann-Gymnasium, Viersen, with grade 8 students (ages 13-14 years). The learners were videographed during their work with the set of cards and asked questions about their way of thinking and their experience with the tasks. In March 2015 four more student interviews were recorded on video at Gesamtschule Bockmühle, Essen. This time grade 8 students (ages 13-15 years) were working in pairs. In addition to these implementations of the tool, there was an expert review of the material in April 2015 in the context of a two-hour research colloquium at the University of Duisburg-Essen. Hereby, the tool was presented and 23 colleagues from the department of didactics of the faculty of mathematics worked on reviewing different aspects of the tool in four groups with different foci. Additionally, the researchers of the German FaSMEd team reviewed the tool with teachers from different schools during the cluster meetings.

The first interviews showed that the tool does allow individual learning paths. A student is able to go through a cycle of solving a task, assessing his/her answer, deciding on which steps to take next and re-evaluating his/her learning progress. Therefore, processes of formative self-assessment can be reconstructed when students work with the developed tool. This can be seen, for example, in the case of Robert's interview that is described in an article by Hana Ruchniewicz (2015, in press).

Re-design of the self-assessment tool
Based on the experiences of the tests and review of the first pen-and-paper version of the self-assessment tool, it was enhanced in the following ways: First, the types of cards were extended (now there are separate types of cards for the test task and the self-check) and their labelling was simplified. In addition, the *Good-to-know*-cards were renamed *Info*-cards in order to simplify the card's structure and due to limited space on the *Check*-card. The new types of cards are:

Test (labelled with a magnifying glass),
Check (labelled with a green and a red check mark)
Info (labelled with a light bulb and as I1-I6),
Practice (labelled with a notebook and as P1-P8),
Expand (labelled with gearwheels and the letter E).

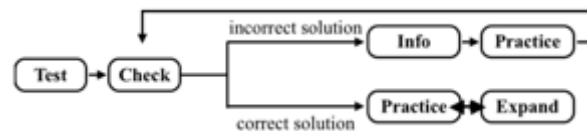Then the hyperlink structure was altered to the following:

Figure 2: Structure of the second version of the self-assessment tool

A student still begins his/her work by solving the open assessment task on the *Test*-card. He/she then looks at a solution on the back of this card and moves on to the *Check*-card. If an error was identified, the student is referred to an *Info*-card in order to revisit information on this mistake and then continues with a matching *Practice*-card. Afterwards, the student goes back to the *Check* (He/she had to go back to the *Test*-task in the previous version, but we found that students only copied the sample solution after seeing it for the first time. This is why, the *Test*-task is not repeated in the new version of the tool.). If the sketched graph is stated as correct by the learner, he/she is forwarded to two more practices and an expand task with a more complex context. These can be worked on in any order as wished by the learner.

The described situation of the *Test*-task was altered as well. Previously the text started with: "Niklas goes for a bike ride that starts at his home. He drives along a street that has no slope at first and then carves up a hill." This has been changed to: "Niklas gets on his bike and starts a ride from his home. Then he rides along the street with constant speed before it carves up a hill." As students were irritated by not having exact values for the speed in the first tests, these changes were made in order to make the situation more explicit while leaving the task open to many solutions. In addition, the naming of the axes was removed in the figure of a coordinate system on the *Test*-card because students are supposed to name the axes by themselves. This way checking their understanding of the relation between the dependent and independent quantity.

Another change is the presentation of the sample solution on the back of the *Test*-card. Previously there were two pictures of possible solutions and a list of criteria for successfully solving this task, labelled with the numbers 1-6. In the test, some students have mistaken this list of criteria with the check-points on the next card. Therefore, the sample solution now consists of one sample graph with different colouring of the parts of the graph that a) reach the value of zero, b) are increasing, c) are decreasing and d) remain constant with matching explanations that state when these cases occur in the story of Niklas' bike ride.

The most important adjustment is the enhancement of the *Check*-card. The 10 check-points have been summarized in 6 new check-points that are more general and do not only refer to the situation described in the *Test*-task. Furthermore, the check-points are now formulated in a positive way, e.g. "I realized when the graph is increasing, decreasing, or remains constant." There is a green and a red check mark for every point of the check-list so that students have to decide for each statement weather it is true or false for their graph. The new phrases of the check-list will result in students checking off a point by marking the green check mark, if they did something correctly. In the first version of the tool, it proved hard for students to check off a point as identifying a mistake. The new check-list will allow students to prove that their solution is right, if they checked off all of the points in the check-list.

Furthermore, the test showed that the previous *Good-to-know*-cards were not helping students while solving the *Practice*-tasks because they referred only to the situation in the *Test*-task. A transfer from this situation to a new situation in the practices seemed to be too difficult. This is why the *Info*-cards now consist of a general information that is meant to serve as a self-repeating unit to help overcome the students' misconceptions, an example that explains the information in the context of the *Test*-task and a figure to visualize the explained example. Moreover, the back of the new *Info*-cards was left empty instead of putting two information units on one card in order to make it easier for students to navigate through the hyperlink structure of the tool.

According to the changes made in the *Check*, the cards *Info 3* and *Practice 3* were added to the tool.

Moreover, the *Practices 1, 2, 5 and 7a* now ask the students to explain their choices or give reasons for their solutions in order to initiate the learners to think about their understanding. The format of the task *Practice 1* was changed into checking off statements so as to avoid students to misunderstand this task. In *Practice 4b* the example was extended by the phrase "In the 2. section …". This was necessary as some students misunderstood the three blank spaces to fill in for section 1 ("In the 1. section, the skier runs _____ (uphill/ downhill) and is therefore going _____ (faster/ slower). The speed _____ (increases/decreases).") as one blank space for each of the three sections of the ski run instead of the description for only the first section. In addition, *Practice 4c* was rephrased into "What is the speed of the skier after 0 seconds, 4 seconds and 8 seconds?" as students were not familiar with the previously more technical expression: "What is the speed of the skier at these points in time: $t=0$, $t=4$ and $t=8$?". *Practice 6* was altered so that students have to choose the dependent and independent quantities for different situations from a list of 8 different quantities. This makes it easier for the learners to compare their solution with the sample solution as a correct answer is no longer an issue of different names when the same quantity is meant. In *Practice 8* the context was changed from a fisherman throwing his fishing-rod into the water to a golfer striking an Ace. The context of playing a ball game is closer to the students' experience in everyday life. Furthermore, the distance of the golf ball from the Tee is more distinct as the distance of the fishing-rod from the end of the pier since one can argue that the fisherman first swings the fishing-rod back over his shoulder before throwing it forward into the water.
Finally, the solutions to *Practices 1, 2, 5, 6 and 7a* have been presented more clearly in form of tables.

(See the second pen-and-paper version of the tool in the appendix of this document.)

Implementation of technology – development of the digital self-assessment tool
Besides the aim of the FaSMEd project to introduce and investigate technology enhanced formative assessment practices, there were several reasons to implement the self-assessment tool in a digital environment. First, the technology offers the opportunity to follow the tool's hyperlink structure in a more intuitive way and by using only one devise instead of a whole set of different cards. Furthermore, a digital tool provides the chance to use dynamic representations and therefore a way to explore the mathematical content of functions more easily than in a pen-and-paper version. Finally, students are motivated by the use of technological devices and their ability to store and process data on the learner's work can help both students and teachers in their formative assessment practices.

Two different types of technology were used to create prototype versions of the tool between September 2014 and May 2015: JACK, a server-based system for computer aided assessment that was developed by the institute of computer science and business information systems at the University of Duisburg-Essen, and the software TI-Nspire Navigator by Texas Instruments. The JACK prototype implements the tool's hyperlink structure and allows the verification of student answers as well as various methods to generate feedback automatically. A benefit of JACK is that exercises with variable contents can be included and that it creates statistics for submitted student solutions, which can be exported to Excel and viewed by teachers and researchers post the students' self-assessment. In addition, it is offered free of charge for the FaSMEd project. Nevertheless, the hyperlink structure of the tool is limited when using JACK because the *Check*-list can not be included fully at once, but only as a single check-point at a time. Moreover, JACK is limited to the following types of exercises: *Fill-In* tasks, in which students give their answers by filling in blank text boxes or tables as well as choosing options from a dropdown menu, and *Multiple Choice* tasks, in which students can mark off answers from a list or a table of possible solutions. Therefore, offering no option of drawing graphs digitally. This is why, students have to solve the *Test*-task in the JACK

prototype by choosing a sequence of different graph-tiles in order to build their graph without being able to move the tiles or put them together accordingly (see figure 3).

Figure 3: *Test*-task in the JACK prototype of the digital self-assessment tool

The TI-Nspire Navigator prototype of the tool took a much longer time to develop. At first, the German FaSMEd team programmed the tasks of the tool with the teacher's software of TI-Nspire Navigator. The programme offers the benefits of allowing the implementation of dynamic illustrations, the generation of statistics from student solutions and the options of working either on a computer, on handhelds or on iPads. Nevertheless, the difficulties of limited question types and the lack of possibilities to insert a hyperlink structure or graphing tool contrained progress of the TI-Nspire prototype for several months. In March 2015, the German FaSMEd team began collaborating with Steve Arnold from Texas Instrument. Steve started to programme the tool via Lua-script and inserting a graphing tool as well as check-list, making the implementation of the tool's hyperlink structure possible digitally (see figure 4).

Figure 4: *Test*-task and *Check*-list in the first draft of the TI-Nspire prototype

After testing and evaluating both prototype versions of the digital self-assessment tool with four FaSMEd teachers, the German F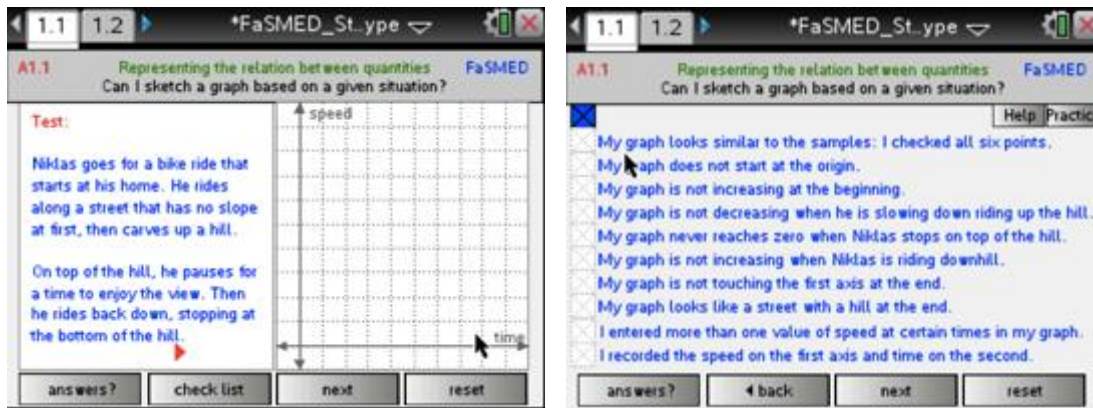aSMEd team decided to only persue its work on the TI-Nspire version. This decision lead to a development process in close collaboration with Steve Arnold. In August 2015, a first test with a year 10 class from Gesamtschule Bockmühle was run using the english version of the tool in combination with translations, that were given to the students on printed screenshots of the iPad display. The reason for this test was to receive first impressions of the tool's handling and identify bugs in the programme. The digital self-assessment tool was finalized in December 2015.

Desciption of the digital self-assessment tool
The digital self-assessment tool includes the same components as the second pen-and-paper version of the tool, namely:

Test (labelled with a magnifying glass),
Check (labelled with a check marks)
Info (labelled with a light bulb),
Practice (labelled with a notebook),
Expand (labelled with gearwheels).

The hyperlink structure of the digital tool alters a little from the second pen-and-paper version (see figure 5):
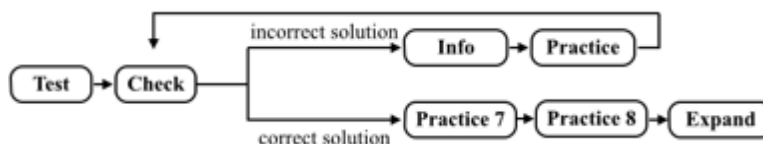
Figure 5: Structure of the digital the self-assessment tool (TI-Nspire)

While the students work on the tool in the same way as they would in the second pen-and-paper version when they identify a mistake in the *Test*-task, the order of working on the tasks *Practice 7*, *Practice 8* and *Expand* is given in the digital tool. This change was made due to the limited space of the digital *Check*-list.

The digital digital self-assessment tool includes four different task types:

- *Graphing*: Students sketch a graph by dragging movable and adjustable graph-tiles into a graphing window. The axes of the coordinate system are labelled by choosing between different options from a drop-down menu. (Used for these tasks: Test, Practice 3a, Practice 6, Practice 7b, Practice 8 and Expand)

- *Open answer*: Students can type in an answer into a text box. (Used for these tasks: Practice 3b and 3c)

- *Selection*: Students answer by selecting the situations for which a certain question is true by double-clicking on buttons that match the situations numbered labels. (Used for these tasks: Practice 1 and Practice 5)

- *Matching*: Students match situations with graphs by first selecting a situation when they click on the button matching the situation's number and then clicking on a graph. (Used for these tasks: Practice 2, Practice 4a and Practice 7a)
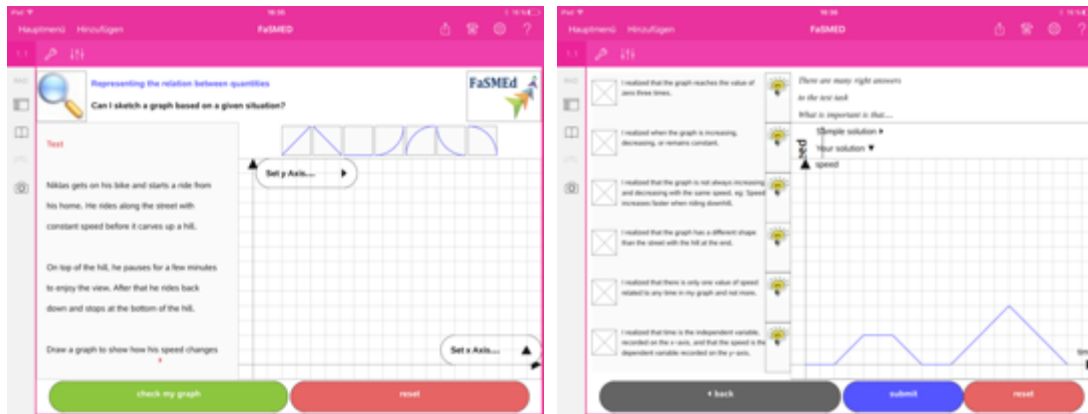
Figure 6: *Test*-task and *Check*-list of the digital self-assessment tool
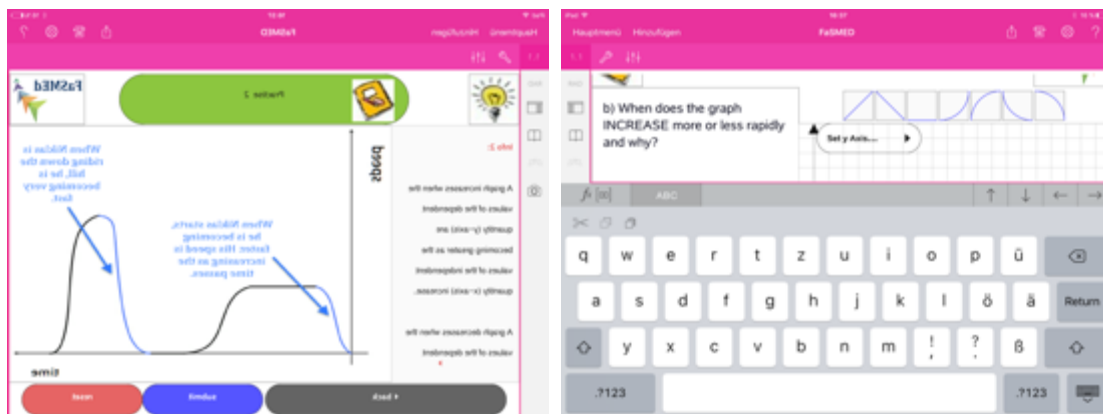
Figure 7: Examples for an *Info* and an *open answer task* in the digital self-assessment tool
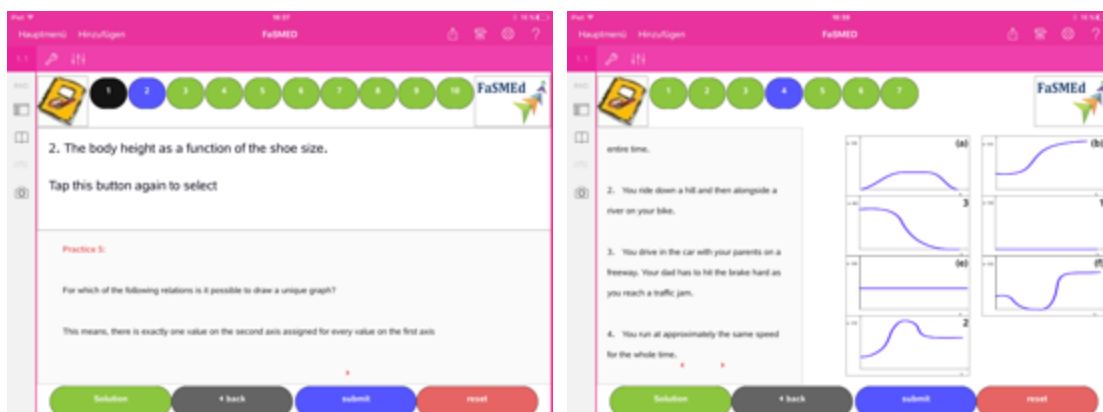
Figure 8: Examples for a *selction* and a *matching task* in the digital self-assessment tool

A language selection between German and English was introduced to the tool in order to make its use easy in German schools as well as for FaSMEd partners. Moreover, by inserting „submit"-buttons for every excercise of the tool, the software is able to store student answers and copy it into a class list in the form of an Excel spread sheet in google drive for the teacher

to review his/her students' work. Steve Arnold set up a webpage to instruct teachers on how to generate this document and introduce the digital self-assessment tool to them via written explanaitions and video tutorials (see http://compasstech.com.au/FaSMED/).

Besides being able to introduce these new features to the digital tool, there are some adjustments that had to be made in comparison to the second pen-and-paper version of the tool: Since it is not easy to change the colour of the font in the digital tool, the colours used to highlight information in the sample solution of the *Test*-task and information have been removed. The *Check*-list entails only one instead of two check marks in the TI-Nspire version because of the limited space available on screen. In a number of tasks students are asked to give reasons for their solutions in the pen-and-paper version (*Practices 1*, *2*, *5* and *7a*). This is missing in the digital version because open answers make it complicated for students to check weather their answers are true or false due to limited screen space. For the same reason, it is not possible to view the solutions in form of a table as in most of the cards. In the digital tool, the sample solution to a task is presented first, before displaying a feedback of which answer the student gave. Finally, *Practice 4d* has been removed because of the difficulty of programming a task, that asks students to rethink their first choice of selecting a graph for the ski-run, has not been possible.

(See the TI-Nspire Navigator file FaSMEd.tns to view the digital self-assessment tool.)

Intended implementation of the digital self-assessment tool
The digital self-assessment tool "Can I sketch a graph based on a given situation?" is designed for students from grade 8 (when they are first introduced to the topic of functions and graphs) till the upper secondary level. It can be used whenever the teacher or student feels the need to repeat this topic and assess the student's competencies of sketching a graph based on a given situation. It is not intended to be used in order to introduce or first learn about functions and graphs. The student works individually and assesses his/her own competencies. Furthermore, the learner decides on his/her own which information to view and which practice tasks to work on based on the *Check*-list. It is best used with an iPad since it has a larger screen (compared to the computer software window or handheld display).

Although the tool is designed for self-assessment, there are other methods to purposefully use the tool in a classroom. As the student's work can be captured by the tool and shown to the teacher in a google drive spread sheet, the teacher has the opportunity to assess the learners' understanding after their self-assessment. Therefore, making it possible for the teacher to address further difficulties in upcoming lessons. Moreover, the tool offers the same structure as an assessment lesson (for example the "interpreting distance-time graphs" lesson of the MARS project). The *Test*-task functions as the initial assessment task and the *Check*-list includes possible issues that students might have with the content. The *Info* units and *Practice* tasks give the teacher suggestions for questions and prompts to address the students' misconceptions. Thereby making it easy for teachers to adjust the tool to be used in a teacher or peer-assessment lesson.

2- "Work with teachers"

The work sessions with the teachers were organized by the German FaSMEd team at the University of Duisburg-Essen or during school visits to different schools. There was one more general meeting with a number of teachers from different schools and more specific meetings that were mostly subject and school specific with only one or two teachers at a time. For a first introduction of the FaSMEd project, eligible schools and teachers were contacted via Email and invited to join a workshop at the university about formative assessment. This preliminary meeting held place in November 2014 and took approximately two hours. Ten teachers, both teaching mathematics and/or science classes, from five different schools were present. The meeting started with a presentation held by the FaSMEd researchers that included theoretical background information on formative assessment, the FaSMEd project, design based research as well as what to expect from the work in the project. In the second part of the meeting, the FaSMEd materials (pen-and-paper version of the self-assessment tool and

translated assessment lesson "time-distance graphs" for mathematics) were introduced to the teachers and they had time to review and comment on the materials. The possible use in their classrooms and implementation of technology was discussed afterwards. All participating teachers agreed to participate in FaSMEd.

Unfortunately, the work with the teachers proofed more difficult over the next stage of the project than expected after the first meeting. This was on one hand caused by the long development process of the self-assessment tool and on the other hand by the teachers not reporting back to the researchers or attending meetings due to time issues. Nevertheless, the researchers found several teachers to participate in the project whenever a new version of the tool needed to be discussed or tested in classrooms, but then meetings were held with only one or two teachers at a time. These covered introducing FaSMEd and its aims, the topic of formative assessment and a review of the tool. In addition, meetings with the teachers were held after classroom trials to discuss the student's work on the tool and its potential as well as suggestions for improvements.

A very important meeting took place in July 2015 at the University of Duisburg-Essen. Three teachers from three different schools met the FaSMEd researchers to review both digital versions of the self-assessment tool (JACK and TI-Nspire). After measuring all advantages and disadvantages of these versions, it was decided unanimously to continue the development only with the TI-Nspire version of the tool. The work on the JACK version was not progressed because its limitations to task types, the hyperlink structure and the handling of the tool as well as the fact, that JACK can only operate when connected to the internet.

One of those teachers was Lars Schweinitz from Gesamtschule Bockmühle. He started participating in FasMEd in August 2014 and continued to assist and review the tool at different stages of the development (see the description above). This is why, we chose to interview Lars about his experiences with the tool and the project for this case study although we couldn't use one of his classes, because his year 10 class had already tested a previous version of the digital tool in August 2015 and his year 8 class did not start to work on the topic of functions and graphs until January 2016. The interview questions were send to him in form of a questionnaire via Email and explained over the phone before he answered them.

Lars reported that the meetings with FaSMEd researchers were in a pleasant atmosphere, motivating and constructive as the work with the FaSMEd materials directed his attention to typical misconceptions. This lead to the teacher's assessments, when observing his students' work, to be more purposeful in the classroom. He reports that working in the project had influenced the way he creates exams and reacts to the gathered information. Lars started to choose tasks that test his students' understanding of a content rather than applying a certain strategy for calculations. He reasons that he is more aware of which tasks to choose for an assessment and that feedback and practice tasks should meet the individual student's needs in order to be more efficient. This is why he sees more meaning in exams as being an assessment of his students' learning. Lars states that after an exam, his work in the classroom can now be more targeted towards the learning goals as he hands a feedback to each student after an exam. This feedback consists of information of what the students need to improve as well as hints on which tasks to work on and is followed by a self-evaluation. He demands his students to ask him for help, when they still have problems after this individual work (see teacher interview).

3- Classroom teaching

Lars Schweinitz teaches the subjects mathematics and technical studies since 5 years. He has been teaching at Gesamtschule Bockmühle, Essen, for 4 ½ years. He is in the age group of 41-50 years and his highest academic qualification is a university diploma in Electrical Engineering. This means that he does not originally come from an educational background, but started teaching as a career changer. In the school, he takes on the role of technical supporter. Besides coming from a technical background in engineering, Lars has worked with different forms of technology in his classrooms. In mathematics he uses spread sheets (Excel) and a geometrical software (GeoGebra) and in technical studies he uses technology for programming and controlling technical processes and microcontrollers. But he had

little experience of working with formative assessment before getting involved in the FaSMEd project after learning about it in May 2014 at a gathering of the Cornelsen publishing house, where Bärbel Barzel of the German FaSMEd team introduced the project to a number of teachers and other stakeholders. Lars joined the project because he finds the idea of assessing students' deficiencies more selective and allowing learners to work on overcoming their difficulties more self-dependent interesting. Especially in his school context, where student achievements are very heterogeneous, he feels that teachers need concepts to teach as efficiently as possible. With that in mind, Lars supports the idea of assessing students based on typical mistakes in order to identify and overcome their misconceptions. In his opinion, the self-assessment tool is able to support teachers as it not only identifies the learners' weaknesses, but includes specific information and tasks to overcome them. Previously to FaSMEd, he participated in another research project at the University of Duisburg-Essen named BiSS (German acronym for: education through writing and speech).

Lars stresses the importance of student orientation, using a variety of methods and fostering the learner's self-dependence in his teaching. He thinks of student collaborations, peer tutoring and the think-pair-share method as effective. For his students it is important to be treated fairly and to understand the learning intentions. They have difficulties to use different mathematical terms in similar topics, for example in the field of percentages and interests, and to think in abstract ways. He regrets that his students do not seem to care about truly understanding mathematical contents but wish for clear instructions on how to solve a task. He attends heterogeneity in his classes by giving students tasks with at least two different levels of difficulty. The students choose themselves, which level to work on. If learners make mistakes, Lars tries to stress that they are normal and important during a learning process and addresses them in class.

When asked to describe formative assessment in his own words, Lars said that formative assessment is "a standardised form of diagnosis/assessment, which enables the teacher to give specific feedback to the students in order to plan the next steps of work more precise/ individualised and therefore more effective."

Since his involvement with FaSMEd, Lars used the first version of the pen-and-paper version of the self-assessment cards in a year 7 class (ages 12-14 years) and a year 8 class (ages 13-15 years) as well as a first draft of the digital self-assessment tool (see description of the tool's development) in another year 10 class (ages 17-19 years). The advantages he sees in students working with the tool are that each learner can work in his/her own pace, self-dependency is demanded and fostered as well as the teacher is exonerated because the learners look for help and solutions themselves. Therefore, it is possible for the teacher to use this free time for observing students' work. In addition, Lars sees a high motivation for the students to work with new technology (iPads). He names the amount of material in the pen-and-paper version (14 different cards), the need of suitable hardware for the digital version (iPads) and the fact that the teacher cannot record all of the students' ideas without making the tool too unclear as disadvantages. Furthermore, he states that students might be overburdened with the demand to assess themselves, so that time is needed to introduce the formative self-assessment tool in length.

Lars found that his students reacted well to the digital self-assessment tool overall and were highly engaged during the lessons, but it was challenging for them to look for help within the material instead of asking him directly. He noticed that well-organised learners worked on the tool very focused and efficiently. The learners used their opportunity to work in their own pace and to engage in subareas of the topic of functions and graphs, which they have not yet mastered. Students, who are not well-organised, have difficulties to understand the tool's structure and start working on each task without confiding in their self-assessment with the *Check*-list. He states that it takes a lot of discipline by the students to work on the tool individually and that students of little interest in mathematics are not more interested by using the self-assessment tool. Nevertheless, Lars mentions that it might take a longer time for students to realize that the tool can help them with their learning. Furthermore, he recognizes the challenge of using this technology for formative assessments, because the tool needs to allow all possible student answers and anticipate possible mistakes. He found it to be effective for students to work on the tool and then group students with the same difficulties together in smaller study groups so that peers can help each other by explaining unclear concepts.

He names a number of affordances, that need to be met for him to use a digital self-assessment tool with his students. First, the tool must consider different linguistic skills among students and the learners have to be able to recognize and document their learning progress immediately. What is more, students have to acknowledge learning as a personal contribution, because they are responsible for the learning process and not the teacher, while using the self-assessment tool. Moreover, there has to be enough material for students to purposefully continuing their work after the assessment and this material has to be designed so that the learners are able to choose the next steps themselves. Lars, sees the use of a digital self-assessment tool restricted by only a small positive effect if used occasionally and the fact that teachers need to change their teaching sequences in order to establish a space for formative self-assessment.

In order to foster his students, Lars uses a learning plan, that students work on individually. This material differentiates student achievements through the students' own work pace and by offering compulsive as well as optional tasks, but not by typical misconceptions as in the digital self-assessment tool. The actual assessment is done by the teacher after observing the students' work over a longer period of time. He states, that this method is chosen by him because it gives him the opportunity to give individualized feedback to the students and due to a lack of accurately fitting assessment material in many mathematical topics. Lars wishes to reorganize his lessons in a longer term in order to foster his students more individually. He only uses the learning plans until more individualized materials for formative self-assessment are available (also in other topics). He wants to integrate formative assessment more permanently in his lessons in order to manage periods of practice according to students' needs. Lars will continue to use the digital self-assessment tool in future lessons and would recommend it to other colleagues and for PD-courses.

(See the teacher interview in the appendix of this document.)

4- Lessons

Context

The self-assessment lesson was taught by two teachers in two different schools in year 10 classes (ages 16-18 years). The first school is Heinrich-Pattberg-Realschule and the second one is Geschwister Scholl Gesamtschule both in Moers, Germany. For this case study we chose the class from Heinrich-Pattberg-Realschule as the unit of analysis because of two single student interviews and a group student interview that were recorded in this class. Heinrich-Pattberg-Realschule is a lower secondary school teaching students from year 5 till 10 (usually ages 10-18 years). There are 45 teachers working at the school and attended by female and male students. It is located in the city of Moers in an urban area and has a very mixed ethnicity among learners ranging between German, Italian, Arabic, Eastern European and many more. There are 10 special needs children in the school. The other students have mixed abilities (although high achievers are not likely to be enrolled in this school, due to the German school system these children often visit a gymnasium). There is very little experience with formative assessment in the school and teachers seemed to focus on test grades and preparing their students for the state-wide final examination at the end of year 10. The use of technologies and technological tools was not common in the mathematics lessons in this school previous to the case study besides the use of overhead projectors and scientific calculators.

The year 10 class consists of 28 students in the age range of 16-18 years. There are mixed ethnicities in the class ranging from German to Eastern European to Arabic. The students are of mixed abilities (few are revisiting year 10) but there are no special needs students in the class. The learners are taught in this class in all of their subjects and generally work well together. While the higher achieving students like to share, discuss and help each other, the lower achievers are more reluctant in the classroom.

The class of Heinrich-Pattberg-Realschule has 60-minute-lessons and a total of 28 students. Because the German FaSMEd team had only 16 iPads to use for the case study, it was decided that students worked in pairs in this lesson. Nevertheless, two students were filmed and interviewed individually during their work with the tool outside of the classroom. The lesson took place on December 21st 2015 from 8-9am and was not implemented in a series of lessons. The students did not review the topic of

functions prior to the lesson in order to see what they still know about graphing and see if the tool helps them assess their misconceptions without any previous interventions. The lesson started with a short introduction to the tool's aims, functions and handling. Screenshots of the iPad were displayed in the classroom with an overhead projector for the introduction because there was no document camera or other projecting devices available in the school. After the introduction, students worked in pairs on the tool for about 45 minutes and were observed by a university student, while two students worked individually outside of the classroom and were interviewed and filmed by Hana Ruchniewicz from the German FaSMEd team. The lesson ended with a classroom discussion about the students' perceptions of the tool.

A priori analysis of the digital self-assessment tool
The mathematical content of the digital self-assessment tool is the transfer of a situational description of a functional relation into a graphical representation. During the development of the tool and especially the *Check*-list, possible conceptual difficulties in this mathematical field were considered. When using the graph of a function to represent the relation between two quantities, three different *Grundvorstellungen* (This German phrase is used to describe links between mathematical contents, real-world contexts and one's individual mental concepts. They represent the meaning of the content and what is essential for its learning.) are important: mapping, covariation and the function in means of a new object (Blum, 2003). A graph can be seen as a collection of points that originate from mapping values of one quantity to another. A more dynamic view is engaged, when the focus lies on the change of two quantities with each other. Then, a graph represents the co-variation of the quantities. Finally, a function can be recognized as a whole object. In order to understand and apply the mathematical term function correctly, students need to establish all of these concepts (vom Hofe, 2003). This is why, the three *Grundvorstellungen* were taken into account in the development of the digital self-assessment tool. For example, *Info 1* and *Practice 1* stress the aspect of mapping because they direct the students' thought to all points of a graph in which the dependent quantity reaches the value of zero. The aspect of covariation is focused, for instance, in *Info 2* where it is explained that *"a graph increases when the values of the dependent quantity are becoming greater as the values of the independent quantity increase"*. The students' issues with the mathematical concept of functions are expressed in their mistakes. The literature shows a large number of typical errors in this field, for example treating the graph as a picture of the described situation (Clement, 1985). These were included in the *Check*-list as they offer hints of the learners' misconceptions and served as the foundation for developing suitable *Info* and *Practice* units.

Referring to the FaSMEd framework, the following global analysis of the tool can be made before the case study lesson. It takes into account possible teaching and learning processed that can occur when the digital self-assessment tool is used in a classroom. As **students** work on the tool individually and are the active agents of the assessment, we can first focus on the formative assessment (FA) strategies the learners use. The tool gives them the opportunity to explore the mathematical content of functions by using dynamic representations as well as offering information on typical misconceptions in form of a *Check*-list. This is why, the functionalities of the technology can be classified as **providing an interactive environment**. Within this environment, students can use four of the FA strategies: First, they are able to **understand the learning intentions** by being presented with the question "Can I sketch a graph based on a given situation?" What is more, the check-list provides them with information on typical misconceptions. This gives students the possibility to recognize **criteria for success** in terms of solving the *Test*-task. Furthermore, the students work on different tasks and need to compare their own solutions to a sample solution. This is how they **elicit evidence about their own understanding**. After solving a task or thinking about a certain check-point, the students need to decide themselves, which step to take next in order to move their learning forward. Therefore, the learner is encouraged in **providing (self-) feedback that moves his/her learning forward**. Finally, the student is **activated as the owner of his/her own learning** in the sense that he/she is challenged to think about their own learning on a metacognitive level as the tool does not advise them on which mistakes were made or what steps to take next. The learner investigates his/her own (mis-)conceptions and adopts responsibility for his/her learning. Thus, we can highlight the following four cuboids in the FaSMEd framework:
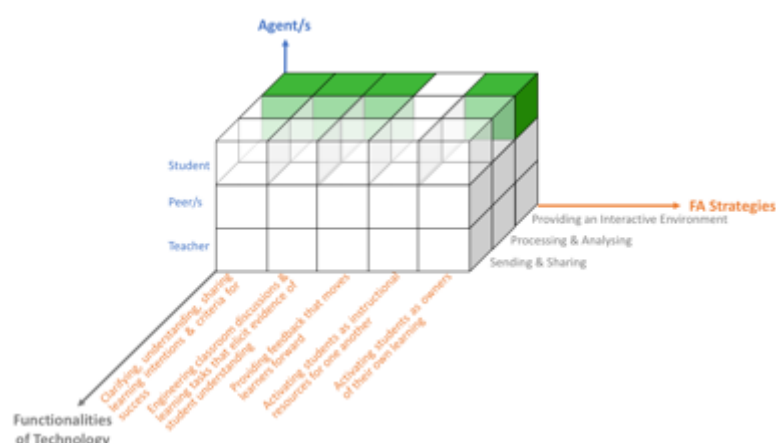
Figure 9: Possible FA strategies used by students working with the digital self-assessment tool functioning as an interactive environment

Accordingly, we can analyse the FA strategies and role of the technology, when focussing on the *teacher*. As he/she is selecting the tool and providing the students with it, we can argue that in this actions the teacher is using the FA strategies *clarifying learning goals and criteria for success*, offering learners *tasks that elicit evidence on student understanding* as well as *activating students as owners of their own learning* in terms of the teacher placing the responsibility for the learning process on the student. In this case, the teacher is using the technology in means to communicate with the students. Therefore, the functionality of the digital tool can be identified as *sending & sharing*. In addition, the digital self-assessment tool stores the student's answers to the test, practice and expand tasks and collects information on which parts of the tool a certain student worked on. This data is being stored in an excel table via google drive, so that the teacher can review the whole class' solutions or one student's work in detail. If this information is used to adopt his/her instructions in lessons to follow the work with the digital self-assessment tool and to advice students on how to move their learning forward, the teacher will be using the FA strategy *providing feedback that moves learners forward*. Here, the tool's functionality is placed in the category *processing & analysing* as the collected data is presented in a statistical way. Focusing on the teacher, we can therefore identify four cuboids within the FaSMEd framework that could be highlighted when working with the digital self-assessment tool:
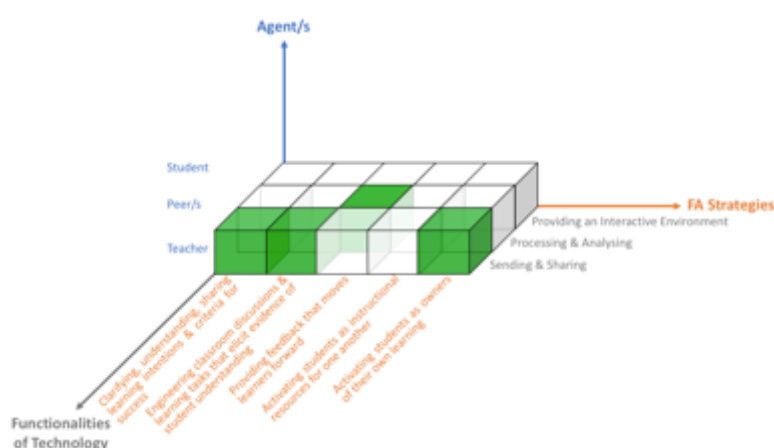
Figure 10: Possible FA strategies used by the teacher working with the digital self-assessment tool in its functionalies of "Sending & Sharing" as well as "Processing & Analysing"

In summary, the global analysis of possible FA processes when using the digital self-assessment tool in a classroom shows that there are rich teaching and learning opportunities in the usage of the material. The highlighted cuboids in the FaSMEd framework on the student level indicate an active involvement of the learner when using the tool for formative self-assessment. Furthermore, a wide range of FA strategies can be activated by the student including the formulation of self-feedback as

well as thinking about one's own learning processes on a metacognitive level. Hereby, the technology functions as an interactive environment providing the learner with the opportunity to explore his/her own understanding. When considering the teacher, other functionalities of the digital tool are in focus. The technology supports the teacher in communicating with his/her students by providing them with information on learning intentions and criteria for success, learning tasks that elicit evidence on the students understanding as well as activating the students as owners of their own learning. In addition, the tool collects and processes data about student achievements when they work in the digital environment. Using this "Processing & Analysing" functionalities of the tool, the teacher is able to adjust his/her instructions or lesson plans for future lessons based on the collected data.
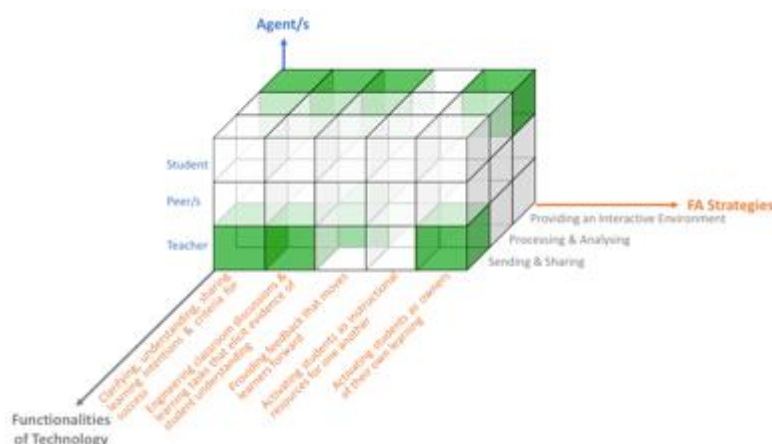

Figure 11: Global analysis of the digital self-assessment tool in regards to the FaSMEd framework

A posteriori analysis of the digital self-assessment tool
For the analysis of the two single student interviews, formative self-assessment processes are reconstructed for each of the learners based on the transcripts of their interviews. Both learners are female and 16 years old. They start by reading the *Test*-task: *"Niklas gets on his bike and starts a ride from his home. Then he rides along the street with constant speed before it carves up a hill. On top of the hill, he pauses for a few minutes to enjoy the view. After that he drives back down and stops at the bottom of the hill. Draw a graph to show how his speed changes as a function of the time."*

Student 1 (S1) solves the task by creating the following graph:
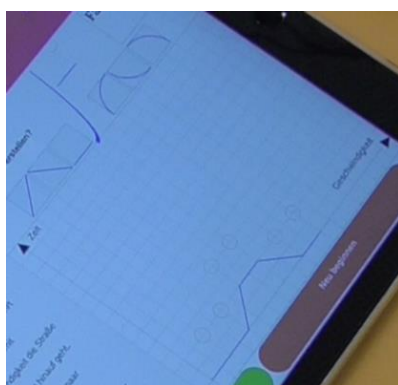
Figure 12: Solution of S1 for the *Test*-task of the digital self-assessment tool

Afterwards, she reads the sample solution and continues to the *Check*. Because she is silent for a longer period, the interviewer asks her, what she is thinking about. The student explains that she is unsure about which *check*-list items to choose because she "saw in the sample solution that there was another graph (with her finger she draws a second hill-shaped graph-segment at the end of her graph) and this was missing in my solution." This shows that S1 assesses her solution by comparing her own graph to the one in the sample solution. Nevertheless, she is unsure about either choosing *Check*-point two or three from the list. This is why, S1 descides to continue with the last *Check*-point *"I realized that the*

time is the independent variable, so it is recorded on the x-axis, and that the speed is the dependent variable recorded on the x-axis." She is sure that this statement is not true for her graph. Then, the student reads *Info 6* and works on *Practice 6*, after the interviewer gives her the hint, that she can work on a practice task concerning this issue if she wishes to do so. In *Practice 6* the students are presented with ten different situations describing the relation between quantities and asked to decide which of the following quantities they have to assign to the axes of a coordinate system if they imagine drawing a graph for the situations: *"temperature, distance, speed, time, pressure, concentration, money and weight."* S1 solves 6 out of 10 items correctly. What it interesting, is that she has no difficulties with situations in which time appears as the independent quantity, but struggles to accept time as being dependent on another quantity. For example, in situation 1 *"In a prepaid contract for cell phones, the time left to make calls depends on the balance (prepaid)."* the learner chooses "time" as the label for the x-axis and "money" as the label for the y-axis.
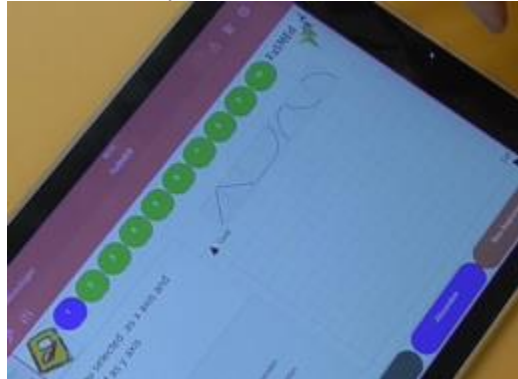

Figure 13: Solution of S1 for *Practice 8 situation (1)* of the digital self-assessment tool

However, she explains "if you have a prepaid phone, you can only make calls as long as you have money." Therefore, she gives the right explanation but still chooses the axes labels in the reverse order. S1 repeats this mistake for several other situations, for instance situation 5 *"Tim's running speed determines the distance he can travel within half an hour."* The student selects "distance" as the label for the x-axis and "speed" as the label for the y-axis, even though she explains correctly that "the speed specifies how far he can run." A possible clarification of this repeating mistake might be found in the way, she proceeds while solving this task. For all situations, she labels the y-axis first and then goes on to the x-axis. Nevertheless, S1 identifies almost all of her mistakes by comparing her solution to the sample solution (except in situation 2, in which she does not see that the y-axis label must be "distance" instead of "speed", and situation 5, in which she does not see that she put the axes labels in reverse order). What is more, she seems to improve during the practice task. While she makes several mistakes at the beginning, she solves the last 5 items correctly. Finally, the student goes back to the *Check*-list and marks off point 6. When asked about an improvement in her understanding, she supports the suspicion that time is a more special quantity to her as she states: "I think I do [understand it better] because I got into the exercise more and more and then also understood more and then I could keep it more in mind with the independent time and dependent quantity."
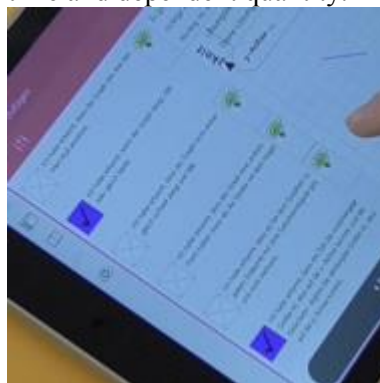

Figure 14: *Check*-list of S1 after working on *Info 8* and *Practice 8* the digital self-assessment tool

In summary, S1 first solves an assessment task, then identifies a mistake, decides to get more information on her mistake, practices the refreshed information and determines her learning success to be sufficient as she marks of the *Check*-point at last. In other words, the students goes through a process of formative self-assessment.

Student 2 (S2) builds the following graph as her answer to the *Test*-task:

Figure 14: *Check*-list of S1 after working on *Info 8* and *Practice 8* the digital self-assessment tool

While she is working on the task, she states: "I think I have to put the time here (points at the x-axis) and the speed here (points to the y-axis)." She labels the axes accordingly in the digital self-assessment tool. After going through the *Check*-points, she does not mark off the sixth statement in the list *"I realized that the time is the independent variable, so it is recorded on the x-axis, and that the speed is the dependent variable recorded on the x-axis."* although S2 labells the axes correctly. When she clicks on the lightning bulb and gets to *Info 6*, S2 reads the text and realizes: "Oh, this is also correct because I did it in the same way." The students then goes back to the *Check*-list and marks off the matching statement. It can be concluded, that by the help of the digital self-assessment tool the student reflects on her work, decides to get more information on a possible mistake and realizes that she did not make a mistake regarding the addressed issue.

Another process of formative self-assessment can be reconstructed by looking at this student's interview regarding the first *Check*-point. She does not mark off this statement and therefore identifies a mistake in her solution to the *Test*-task. Then, S2 decides to learn more about when the graph reaches the value of zero as she continues to *Info 1*. She reads the information and says that she now understands it. When asked, what it is she understands, S2 explains with the help of the figure in *Info 1*: "I did not do it like this, I did it so that Niklas rides along the street (point to the first increasing part of the graph) and then here (points to the first segment of the graph that remains constant) he rides along the hill and then he stops, but I did it so that he goes back again (points to the first decreasing part of the graph). I did not do it with the second zero, when Niklas stands on top of the hill then he has no speed anymore."

Figure 15: *Info 1* of the digital self-assessment tool

After this insight, S2 goes back to the *Check*. The interviewer tells her that if she wants, she can do an exercise on the previous matter as well. This is why, S2 decides to go to *Practice 1* next. In this tasks, students are given the story of Marie walking home from school, for each part of this story, they are asked to decide weather the graph for this situation reaches the value of zero. S2 solves this task correctly for all statements and is only unsure about fourths part of Marie's walk from school: *"After Jana says goodbye, Marie goes on more quickly."* S2 explains that she is unsure about the meaning of "saying goodbye" as she thinks that the girls in the story would have to stop in order to do so, but then the statement says that Marie goes on more quickly. After the interviewer explains that it means that the girls already said goodbye, S2 correctly responses that the graph is not reaching the value of zero in this case. By looking at the sample solution, the student acknowledges that she solved the task correctly. This is another example of a formative self-assessment process because the student first makes a mistake, identifies it, decides to get help regarding her issue, overcomes her problem by reflecting her mistake in the given situation and finally, tests her learning process by solving the practice task and comparing her answer to the sample solution.

These three examples of formative self-assessment processes show the potential of the developed tool. It makes it possible for students to identify their mistakes, give themselves feedback by comparing their own solutions to the *Info* units and sample solutions as well as the *Check*-points and make progress in their learning. What is more, they adopt responsibility for their own learning. Nevertheless, the interviews also show that the students can not identify all of their mistakes. For example, both students do not realize that they have an issue with knowing when a graph is increasing, decreasing or remains constant because both of them mark off the second *Check*-point even though their solutions would not allow this. This is why, the tool is restricted to being an additional method for formative assessment in classrooms and can not be seen as a replacement for the teacher or the students' peers. But it can give students and teachers the opportunity to learn more about the learners' understanding and support the students to be more self-dependent.

(See the transcripts of the single student interviews in the appendix of this document.)

5- Pupil perceptions

The q-sorting and student interview took place at Heinrich-Pattberg-Realschule in the lesson following the students' work with the digital self-assessment tool (9-10am). Four female students (ages 16-17 years) volunteered for the interview and were recorded on video outside of their classroom. The two students who were filmed during their work with the tool individually were not part of this group. It took them approximately 10 minutes to sort 60 statements into the following four categories: *we agree completely, we agree a little*, *we disagree a little*, *we disagree completely* (see figure 9). Afterwards the learners were asked about their sorting choices and perceptions of the digital self-assessment tool based on their q-sorting and the FaSMEd group student interview guideline.

Figure 9: Results of the students' q-sorting activity

Concerning the students' perception of mathematics, the q-sorting shows that while they think it is an important subject that is relevant for everyday life, they find it frustrating and difficult. In the interview, one of them (S3) explains that mathematics is difficult for her because she finds it hard to think logically. The learners think of mathematics as being either right or wrong and requiring a lot of repetition. It is something that everybody is able to learn, but understanding mathematics is dependent on the teacher. The participating students are not confident in their mathematical skills and state that

while they are not nervous in the classroom, they are in examinations. When studying mathematics, they prefer to work in groups with their peers, but also use the textbook they know from the classroom as well. Furthermore, they explain in the interview that it is easier to correct a mistake if the teacher or other students explain to them, what they did wrong. The four learners put the statement "In mathematics lessons there is no room for expressing one's own ideas." in the category "We agree completely". When asked about this match, it becomes clear, that they mean the opposite: if their teacher explains a certain task and they chose to do it a different way, the teacher encourages them to follow their own ways of solving a problem. It shows that the students do not see exams an opportunity to learn, but find them boring. In the interview, S2 explains that she usually knows her weaknesses before the exam. When asked about the statement "Exams help me to work more.", which the group placed in the category "we disagree completely", three of the students explain, that they only practice before an exam and then forget everything afterwards. However, one student (S1) does argue that exams help her study more and provokes the statement "If I do not understand something, I work with it until I get it right." to be sorted in the category "We agree a little".

When looking at the statements regarding technology, it becomes clear that the students have very little experience with its use in mathematics lessons. They only use materials like set squares, overhead projections or scientific calculators in the lessons. While their teacher does have an iPad at hand, she only uses it for organisational work, e.g. to record missing homework or grades. Nevertheless, the students state that they like using technology in mathematics and find it useful. In the interview, they name motivation, easy handling and mobility as the advantages of technology. They explain, that it is more practical to just have one iPad to study with, rather than a book, a pen and an exercise book. This gives the learners the opportunity to study anywhere, for example in the school bus. Furthermore, S1 thinks that using technology in mathematics could help students to become more independent because they have to get the information themselves, rather than having the teacher explain it to them.

In regard to the digital self-assessment tool, the four students rate the q-sorting statements reluctantly. While they place "With the technology I can find out weather I can do the tasks in mathematics." in the category "We agree a little", they "disagree a little" with several other statements concerning the tool, for example "I prefer to talk to the teacher, rather than find out myself with the technology." or "When we use technology during the mathematics lessons, I better understand where my weaknesses lie." A reason for this matching could be the little experience the students have with using technology and assessing themselves. As they seem very insecure about their mathematical achievements, it might overwhelm the students to be responsible for their own learning. S1 explains that she prefers to assess herself with the digital self-assessment tool, but would like a "real solution" afterwards. The need for feedback from a teacher or peers seems essential for her learning. It is interesting that all four students state that working with the tool created a quite atmosphere in the classroom. Finally, they do think it can help them with their learning to do self-assessments more often.

In addition to the q-sorting and group student interview, the observation of the classroom discussion at the end of the lesson give insights of the students' perception of the digital self-assessment tool. Students like the easy handling and to work in their own pace. They state that the *Info* units are helpful, but it is unfamiliar for them assess themselves. Therefore, they wish for more explanations and additional feedback. The student found handling the tool difficult at the beginning but state that working the tool gets easier with time. Furthermore, the learners identify several technical issues. First, the single graph-segments and axes labels in the graphing tasks are not erasable. In order to correct a misplaced graph-tile or make changes to the graph, students need to press the "reset" button and start over again. This is very frustrating to the majority of the class. Moreover, some students point out, that they are confused with the choices of colour for the graph-tiles. When selected and pulled into the graphing field, the tiles appear in a red background. This irritates the learners as they think choosing a certain tile is wrong. Finally, some bugs in the programme were found in which parts of the screen would freeze and cover the next screen.

(See the students' q-sorting results and interview in the appendix of this document.)

II CONCLUSIONS

This case study shows that the developed tool does allow processes of formative self-assessment and improvements in students' understanding of functions. When used in a classroom, it influences both students and teachers. The learners gain motivation, are highly engaged in the learning process and can work in their own pace. Furthermore, they become more independent by adopting responsibility for their own learning. The teacher has more time to observe his students' work and an opportunity to give more individualized feedback to the learners. Furthermore, the engagement of the teacher with the tool's concept can lead to a change in his/her teaching. In the case of Lars, he changed his practice in regards to exams to gather more information on his students' understanding. Nevertheless, the digital self-assessment tool still faces many challenges. The complex structure of the tool makes it hard for students to handle it properly. Therefore, the tool can not simply be handed to the learners, but needs clear and extensive instructions. In addition, they need to accept the responsibility for their own learning and notice the tool as an instrument to move their learning forward. Moreover, students that feel insecure about their mathematical competencies wish for a lot of guidance and feedback from their teacher and peers. These learners need a lot of practice to adjust and grow more confident in assessing themselves. For them it might be better to use the tool in order to match students in groups struggling with the same issue. Finally, the documentation of the students' work with the tool remains a concern. The tool only secures the students' work in a google drive spread sheet if the iPad can be connected to the internet. This can not be assured in all schools. Furthermore, the representation of the results only shows the students' answers, but does not record the info units, questions or the path that the learners follow through the structure of the digital self-assessment tool.

III REFERENCES

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